PHY 101 –General Physics I
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Position, velocity and acceleration vectors

Projectile motion

Motion in a circle etc

Position, Velocity and Acceleration vectors

Displacement can be defined as the vector difference between the initial and the final positions of an object OR

Distance travel in a specific direction (a vector quantity)

- # Position vector

Position vector

$\vec{r}_i = \text{initial position of an object},$

$\vec{r}_f = \text{final position of an object},$ then

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$\vec{r} = \text{Position vector from the origin to the position of the object}$

Velocity vector

If the velocity of an object changes from [illegible] to [illegible] where

[illegible] = [illegible],

[illegible] = [illegible], then

Δ[illegible] = [illegible] − [illegible]

[illegible] = [illegible]

Acceleration vector

If the acceleration of an object changes from [illegible] to [illegible] where

$$[illegible] = [illegible],$$

$$[illegible] = [illegible],$$ then

$$\Delta [illegible] = [illegible] - [illegible]$$

$$[illegible] = [illegible]$$

Instantaneous velocity and acceleration

Instantaneous velocity and acceleration

Recall that the average velocity and average acceleration of the object are given as

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$v_{ins} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ **where x-displacement i.e the velocity of an object at any instant in time (i.e as the limit of Δt→0) is called instantaneous velocity (m/s)**

$a_{ins} = \lim_{\Delta t \to 0} (\Delta v/\Delta t)$ **where v=velocity i.e the acceleration of an object at any instant in time (i.e as the limit of Δt→0) is called instantaneous velocity (m/s^2)**

Velocity and acceleration

Velocity is defined as rate of change of displacement (m/s)

$v(t) = \frac{dx}{dt}$ where x=displacement

$$dx = v(t)dt$$

$The\ displacement,\ x = \int_{t_1}^{t_2} v(t)dt$ from t_1 to t_2

Also, Acceleration is the rate of change of velocity or change in velocity over time taken (m/s^2)

$a(t) = \frac{dv}{dt}$ where v=velocity

$$dv = a(t)dt$$

$The\ displacement,\ v = \int_{t_1}^{t_2} a(t)dt$ evaluated from t_1 to t_2

In 3 Coordinate system,

In 3 coordinate system, If the displacement is given as

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

Calculations

Problem 1

The position of an object moving in a straight line is given by
$x = 3t - 4t^2 + t^3$ *where x is in meters and t is in seconds, calculate (i) the object's displacement between* t=0 and t=4 s
(ii) the average velocity for the time interval t=2 s and t=4 s

Solution

At t=0 s, $X_1=3(0)-4(0)^2+(0)^3=0$ m

At t=4 s, $X_2=3(4)-4(4)^2+4^3=12$ m

The displacement between t=0 and t=4 s =12-0 =12 m

(ii) at t=2 s, $x_1=3(2)-4(2)^2+(2)^3= -2$ m

Recall that at t =4 s , $x_2=12$ m

$$\text{Average velocity} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{12-(-2)}{4-2} = \frac{14}{2} = 7\ \text{m/s}$$

Problem 2

A particle moves along the x-axis such that its instantaneous position x is given as $x = pt^2 - qt^3$ where x is in meters and t is in seconds. If the numerical values of p and q are 2.0 and 1.0 respectively, calculate (i) the dimension of p and q (ii) apart from t=0. At what time iis the particle at rest and determine the acceleration at that time

Solution

Given that $x = pt^2 - qt^3$

Both pt^2 and qt^3 must have the dimension of length L

$L=Pt^2$, $P=L/t^2=LT^{-2}$

$L=qt^3$, $q=L/t3$: $q=LT^{-3}$

(ii) Given that $x = pt^2 - qt^3$ V=0 when the particle is at rest

$$V = \frac{dx}{dt} = \frac{d\,(pt^2 - qt^3)}{dt} = 2pt - 3qt^2$$

$2pt - 3qt^2 = 0$: $t(2p - 3qt) = 0,\; t = 0 \; or \; 2p = 3qt$

$$t = \frac{2p}{3q} = \frac{2x2}{3x1} = 1.33\,s$$

The acceleration, $a = \frac{dv}{dt} = 2p - 6qt = 2(2) - 6(1.33)(1) = -3.98\,m/s^2$

Problem 3

The velocity of a particle moving along the x-axis varies with time according to the expression $V_x = \frac{\left(40 - 5t^2\right)m}{s}$ *where t is seconds. Calculate* the average acceleration in the time interval t=o to t=20 s

Solution

$$a = \frac{dV_x}{dt} = \frac{d}{dt}\left(40 - 5t^2\right) = -10t \quad \text{evaluated from} \begin{matrix} 20 \\ 0 \end{matrix}$$

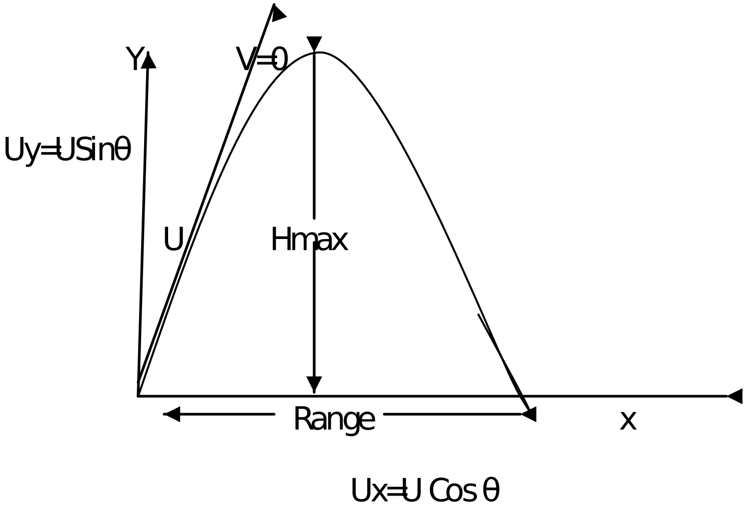
=-10 (20) = -200 m/s^2 (i.e deceleration or negative acceleration)

- TUTORIAL #1

- All Class representatives must see me to collect it.

Projectiles

- What is a projectile? Examples
- What is a trajectory?
- Angle of projection (θ)
- Time to reach the maximum height (t)
- Time of flight (T)
- Range (R)
- Maximum height (Hmax)
- Instantaneous velocity of a projectile (V)
- Calculations


Y
V=0
Uy=USinθ
U
Hmax
Range
x
Ux=U Cos θ

Projectile-Definition and examples

- A projectile is a body that is launched into the space. Which usually follows a curved or parabolic path.
- It is a motion under the influence of ***gravitational acceleration*** and ***air resistance***.
- It has two types of motion. (i) the constant horizontal motion (ii) a vertically downward motion.
- The path taken by the projectile is called trajectory.
- *Examples*-Projectile motion
- -Launching of a mechanically propelled missiles
- Shooting of arrows, gun and rockets
- -Throwing of discuss and javelin
- -taking off and landing of an aircraft
- -Kicking the ball or throwing of a basket ball

Time (t) reach the maximum height (t) and Time of flight (T)

$$V = U_y - gt$$

At H_{max}, , $V=0$ $\quad 0 = uSin\theta - gt$

$$t = \frac{USin\theta}{g} = time\ to\ reach\ H_{max}$$

The time of flight= $\frac{2USin\theta}{g} = 2t\ i.e\ twice\ the\ time\ to\ reach\ H_{max}$

Range (R)

The horizontal distance from the point of projection to a point where the projectile hits the projection plane

$$The\ horizontal\ velocity\ U_x = \frac{horizontal\ distance}{time\ taken} = \frac{R}{T}$$

$$R = U_x \times T = UCos\ \times \frac{2USin\theta}{g} = \frac{U^2 Sin\ 2\theta}{g}$$

Range is maximum when Sin 2θ=1

$2\theta = 90^0$

$\Theta = 45^0$

Maximum height (Hmax)

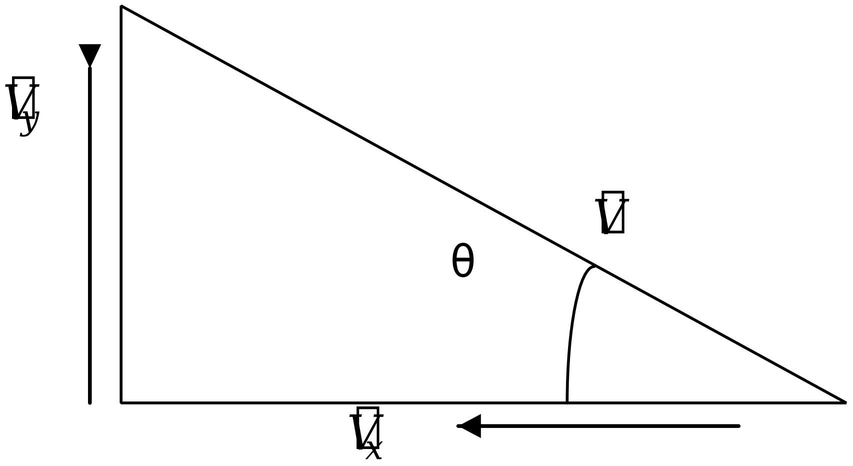
$$Using\ V^2 = U_y{}^2 - 2gS\ ,\ a = -g$$

$$0 = U^2 Sin^2\theta - 2gH_{max}$$

S=Hmax, Hence $,H_{max} = \frac{U^2 Sin^2\theta}{2g}$

Instantaneous velocity

Instantaneous velocity, V



Using Pythagoras Theorem,

$$V_x^2 + V_y^2 = V^2$$

$$The\ instantaneous\ velocity, \vec{V} = \sqrt{V_x^2 + V_y^2}$$

The angle of projection $\theta = tan^{-1}\frac{V_y}{V_x}$

Problem 1

A gun that is inclined at 30^0 to the horizontal fires a bullet with a speed of 500 m/s. Calculate the velocity of the bullet 5s after it is fired ($g=9.8$ m/s^2) and the resultant speed of the bullet?

Solution

$V_x = U_x = UCos\,\theta = 500Cos30^0 = 433$ m/s

$U_y = USin\,\theta = 500Sin30^0 = 250$ m/s

The velocity /vertical component after 5s,

$V_y = U_y - gt = 250 - 9.8\,(5) == 201$ m/s

The resultant speed of the bullet,

$\vec{V} = \sqrt{(433^2+201^2)} = 479.4$ m/s at an angle of projection $\theta = tan^{-1}\frac{V_y}{V_x}$

$$\theta = tan^{-1}\frac{201}{433} = 22^0$$

Problem 2

Problem 2

An object is projected with a velocity of 30 m/s at an angle of 40o to the horizontal.
(i) the time to get to the maximum height

(ii) the maximum height reached

(iii) the total distance covered (g=10m/s^2)

Solution

A=30m/s. θ=40^o

(i) $\square = \frac{\square\square\square\square\square}{\square} = \frac{30\square\square\square\ 40^{\square}}{4} = 1.93\,\square$

(ii) **The maximum height reached**

$$\square\ \square\ \square\square = \frac{\square^{2}\square\square\square^{2}\square}{2\square} = \frac{(3040\square\square\square\square\square)^{2}}{\square\square} - 18.6\,\square$$

(iii) $\square_{\square} = \frac{\square}{2\square}$

$$\square = \square_{\square} . 2\square$$

$$= \square\ \square\square\square\ \square\square$$

$$304021\square\square\square\square\square\square\square.93 = 88.7\,\square$$

TUTORIA #2

- Questions 1-4

Circular motion

- Centripetal force and centripetal acceleration
- Centrifugal force
- Relationship between

(i) Linear velocity and angular velocity

(ii) Linear acceleration and angular acceleration

The angular equation of motion

-Calculations

CIRCULAR MOTION

- This is the motion whose acceleration is directed radialy towards the centre of the circle and it is given in magnitude as $a_c = \frac{V^2}{r}$
- Constant speed
- From Newton's 2nd law of motion, the net radial force (**centripetal force**) is

$$F_c = ma_c = \frac{mv^2}{r}$$

Centrifugal force (tends to pull the object away from the centre)

$$F_g = ma_c = -\frac{mv^2}{r}$$

The force Fc) is always directed to the centre and acts perpendicular to the direction of the linear velocity

Period is the time taken for a particle to complete one revolution around a circular path of radius r is referred to as period (T) of the circular motion

$T = \frac{The\ circumference\ \ of\ \ the\ circle}{speed} = \frac{2\pi r}{v}$ Unit : Seconds (s)

The tangential component of acceleration $a_t = \frac{dv}{dt}$

The linear acceleration is related to centripetal acceleration by

$$a_L = r\square = r\frac{d\square}{dt} \quad where\ \square = angular\ velocity\ (\frac{rad}{s})$$

$$The\ linear\ velocity, \quad V = r\square$$

$a = r\square^2$ Since F =ma . $F = mr\square^2$ (Newton, N)

$$\alpha = angular\ acceleration\ (\frac{rad}{S2})$$

The radial component of linear acceleration is the resultant of a_t amd a_r .

Linear acceleration

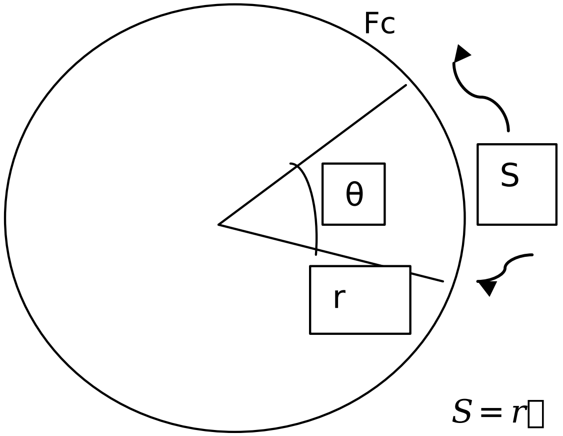
tangential acceleration

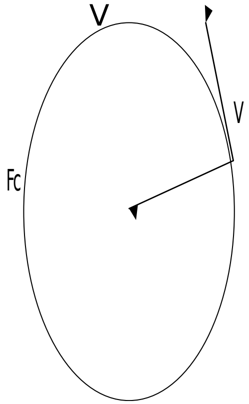
Radial component of acceleration

$$a^2 = a_t^2 + a_r^2$$

$$a = (a_t^2 + a_r^2)^{\frac{1}{2}}$$

$$a = (a_t^2 + a_r^2)^{\frac{1}{2}}$$





$$S = r\theta$$

$$The\ linear\ velocity, V = \frac{S}{t} = \frac{r\theta}{t} = \frac{displacement\ along\ the\ curved\ path}{time\ taken}$$

Angular velocity $\omega = \frac{\theta}{t}$ $\Theta = \omega t$

The angular equations of motion are:

(S=θ, ω0 and ω = initial and angular speed, a=α)

$\omega = \omega_0 + \alpha t$ -------------(1)

$\omega^2 = \omega_o^2 + 2\alpha\theta$..........(ii)

$\theta = \omega_o t + \frac{1}{2}\alpha t^2$............(iii)

Calculations

Problem 1

Suppose the moon is moving in a circular path of radius 3.8x10^5 km about the earth. If the period of revolution is 27.5 days, calculate the acceleration of the moon

Solution

R=3.8 x 10^8 m, T=27.5 days = 24 x 3600 = 2.37 x 10^6 s

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 3.8 \times 10^8}{2.376 \, x \, 10^6} = 1 \, x \, 10^3 \, m/s$$

$$a = \frac{v^2}{r} = \frac{(10^3)^2}{3.8 \, x \, 10^8} = 2.36 \, x \, 10^{-3} \, m/s^2$$

Problem 2

An earth satellite rotates in a circular orbit of radius 7,000 km (about 600 km above the earth’s surface) with an orbital speed of 27,000 km/h. calculate: (i) the period of the rotation (ii) the acceleration due to gravity at the orbit?

Solution

Given that r= 7 x 106 m v=27,000 km/h=27000 x 1000/3600 m/s = 7500 m/s

(i) $T = \frac{2\pi r}{v} = \frac{2\pi \, x \, 7 \, x \, 10^6}{7500} = 5.864 \, s$

(ii) $F = mg_r = \frac{mv^2}{r}$ $g_r = \frac{v^2}{r} = \frac{7500^2}{7 \, x \, 10^6}$ =8.04 m/s^2

Thank you